Vol-3, No.-2, Nov. 2012 PANCHAKOTesSAYS ISSN : 0976-4968

RECTIFYING DEPRIVATION BY THE APPLICATION OF CONVEXITY IN CONSUMER PREFERENCE AND PRODUCTION PROCESS

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Introduction

Optimum allocation of resources has an important role in rectifying deprivation. The mathematical property of convexity is used in various fields of economics. The idea of convexity may be use for optimum allocation of resources and thereby reduce deprivation. Without this property we cannot reach the consumer's equilibrium point. Without this property the demand and supply functions are discontinuous and hence we cannot draw the market equilibrium point. We will not be possible to use the mathematical programming and duality in the absence of convexity (K C Roychowdhury 1991). The convex budget set only imply and interpret the convex indifference curve (IC) and diminishing marginal rate of substitution (MRS). Given the assumption of non-satiety, strict quasi-concavity of the utility function ensures that IC is strictly convex to the origin, or alternatively, the MRS is diminishing. In non-convex preferences the consumer prefers a glass of beer or a glass of champagne alone to any mixture of the two.

Many opined that the main cause of food crisis, poverty and famines were the non-availability of food supply. But in practical experience we saw that even there was no food deficit famines have occurred in many countries due to lack of entitlement to food. The Nobel laureate economist A. K. Sen has given many examples of such famines in his analysis in different countries of the world. He argued that there was enough food in the world to feed everyone adequately but the problem was fair public distribution system (PDS) of food and hoardings by traders to rise in price to make profit. The crisis not food crisis, it is the policy crisis of the government. iii) Food absorption is necessary for food security. It is not sufficient to take food only to satisfy the hunger. It is necessary to see to it that the food taken is absorbed in the digestive system and provides nutrition to the body. For this we need pure and safe drinking water, hygienic environment to live, primary health awareness and provision of basic education to keep the environment clean and healthy.

The idea may be applied to measure the poverty related matters in broader sense. The average consumption combination of any two commodity bundles are preferred than extremes. The idea of monomania is not so a desirable property both for the consumption and resource allocation point of view. The convex idea shows that all resources would not use to produce in a single commodity production simply because the one item cannot serve the all human wants. For an individual three basic essential things are required for his survival: food, clothing and shelter. In the primitive community when man lived in jungles he did not have clothing or even shelter. But he needed food to survive. Animals can live without clothing or shelter but also need food. Plants also need food. While plants can make their own food man and other animals have to produce or collect food. Thus for all living beings food is the most essential component of life. It is necessary for getting energy which man needs for doing different works. Even when a man is sleeping his major organs like heart or lungs remain active. These are functioning from birth to death at a stretch. To continue these activities energy is required which can be derived from food (R Bairagya & J Sarkhel 2011).

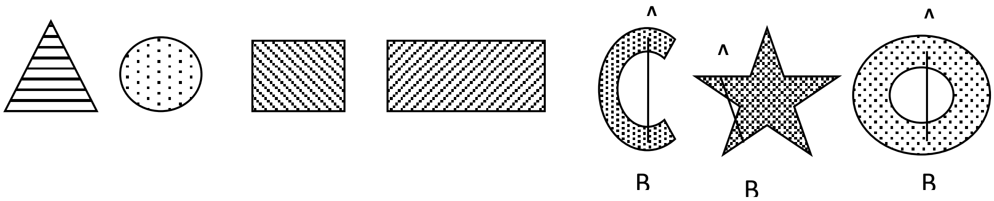
Convex Set

A set is called convex iff any straight line (joining by two points from the set) entirely lie on the set. Suppose x∈A, y∈A, then the joining straight line,

$$\overline{xy} = tx + (1-t)\, y \in A,\ t \in [0, 1]$$

Now tx + (1-t) y is called the convex combination. When t=0, y∈A and t=1 x∈A. An intermediate values of t i.e. $0 < t < 1$ gives us the weighted average of x & y.

The solid triangle, circle, square, rectangle etc are the examples of convex set because any straight line drawn from taking any two points lies entirely on the



set. A crescent shape or a star, or a hollow circle is not convex set. Straight line like $\overline{AB}$ does not lie entirely on the set.

Properties

Any vector is an extreme point of a convex set if it cannot be expressed as a convex combination of two other vectors in the set i.e. an extreme point does not lie on the line segment between any other two vectors in the set.

Any vector is a closed and bounded convex set with a finite number of extreme points can be expressed as a convex combination of the extreme point.

The solution space of a set of simultaneous linear equations is a convex set having a finite number of extreme points (R Bronson and G Naadimuthu 1997).

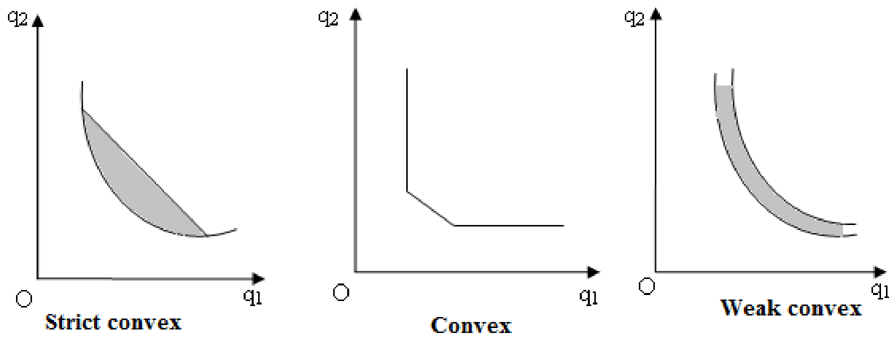
In consumer preference theory we assume that all consumers are rational and behave in accordance with a systematic consistent set of preference. Moreover in consumer choice theory we assume that each commodity is finely divisible so that any non-negative quantity can be purchased by the consumer. The consumption set can be written as:

$C = \{x = (x_1, x_2, x_3, x_4 \ldots x_n), x_i \geq 0, \forall i = 1,2,3, \ldots \ldots, n. \}$

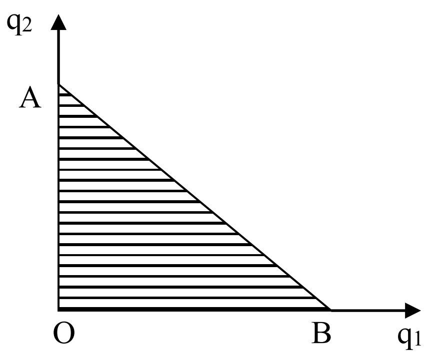
Now the set C is a closed, convex set.

Suppose a consumer is indifferent between two commodity bundles x and y in the set C i.e. xIy, x, y ∈C. Now any weighted average of x & y always prefer to either alone. Thus if xIy, then tx + (1-t) y P x or y $\forall t$, 0<t<1, $x \neq y$

Now strict convexity implies that IC is convex towards the origin but it can contain straight line segments and a weak convex preference allows thick band IC (K C Roychowdhury 1991).

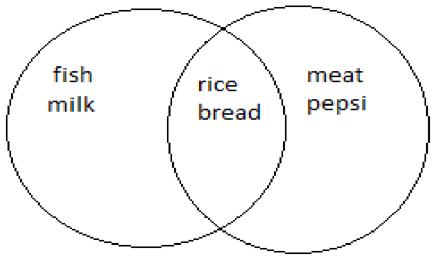


In economics, the commonly used budget set is closed and convex. △ OAB is a convex set. Any point on this set is affordable by the consumer. At point O, the consumer does not buy either of any commodities. The other two extreme points i.e. at A, the consumer purchases only q_2 and at B the consumer purchases only q_1. This is the situation of monomania.



Suppose A & B are two convex sets. Then A intersection B is a convex set but A union B may or may not B a convex sets.

Suppose the consumption bundles of two individuals A & B are:



A = {rice, bread, fish, milk}

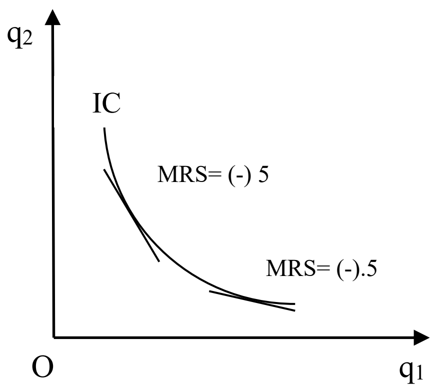
B = {rice, bread, meat, pepsi}

Let the equation of IC be $U = f(q_1, q_2)$.

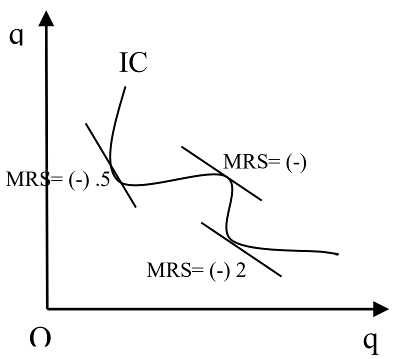
Long the IC, $f_1 dq_1 + f_2 dq_2 = 0$ or

$\frac{dq2}{dq1} = -\frac{f1}{f2} < 0; f1, f2 > 0 \rightarrow$ IC has negatively sloped.

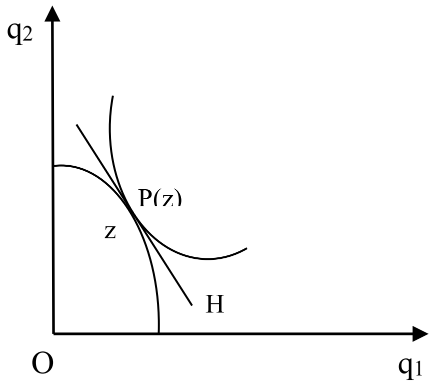
∴ MRS decreases along an IC. MRS decreases in absolute term iff preferences are strictly convex.



MRS is not always decreasing as q_1 increases when the preferences are non-convex.



Any disjoint pair of convex sets can be separated by a hyper plane. Any point in the region to the right of z is preferred.



Convex function

Convex functions are closely related to convex sets. If f(x) is convex, then for any constant, say k, it can give rise to a convex set, say S (AC Chiang 1984).

If S ≡ $\{x|f(x) \leq K\}$ then f(x) is convex.

If S ≡ $\{x|f(x) \geq K\}$ then f(x) is concave.

Suppose Y = f(x) be differentiable and second order derivative exist.

Then f(x) will be strictly convex iff f $f''(x) > 0 \rightarrow$ f(x) has a minimum value.

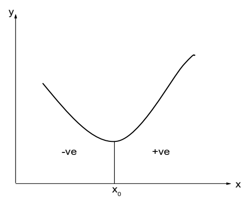
Then f(x) will be strictly concave iff f $f''(x) < 0 \rightarrow$ f(x) has a maximum value.

Suppose Y = f(x) be differentiable and strictly convex and it has a minimum at point x = x_0.

Then $f'(x) < 0$ *for* $x < x_0$

$f'(x) = 0$ *for* $x = x_0$

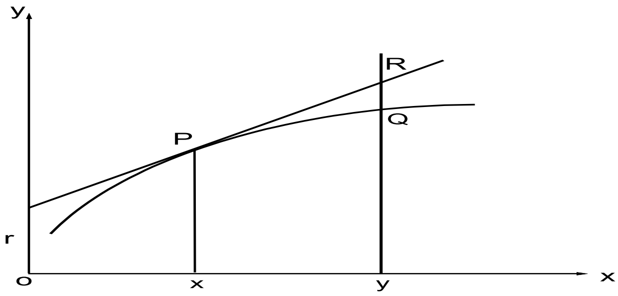
$f'(x) > 0$ *for* $x > x_0$



$f'(x)$ Changes sign from –ve to +ve at point x_0

The situation will be reversed if the function is concave.

Let y = f(x) is concave iff any tangent drawn at point P lie entirely above or on the graph (C Birchenhall and P Grout 1987).



t(y) = r + $f'(x)$y. At the tangency point f(x) = t(x) and x = y

f(x) = r + $f'(x)$x $\rightarrow r = f(x) - f'(x)\,x$

$\therefore t(y) = f(x) - f'(x)x + f'(x)y$

Or $t(y) = f(x) + f'(x)(y - x)$

Now if Q lies below R then f(y)$\leq t(y) = f(x) + f'(x)(y - x)$

Thus y = f(x) is concave iff f(y)$\leq f(x) + f'(x)(y - x)$

And y = f(x) is convex iff f(y)$\geq f(x) + f'(x)(y - x)$

Hessian-Determinant to check convexity:

Let multivariate function, z=f(x, y).
Then f(x, y) will be convex iff $z_{xx}>0$ and $z_{xx}.z_y > (z_{xy})^2$.
And f(x, y) will be concave iff $z_{xx}<0$ and $z_{xx}.z_y > (z_{xy})^2$.
A convenient or sufficient test or second order condition for convexity/concavity is the Hessian Determinant.

$|H| = \begin{vmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{vmatrix}$

When$|H_1| > o \text{ and } |H_2| > 0$, the Hessian is called +ve definite and the function are convex and the function has a minimum value. When$|H_1| < o \text{ and } |H_2| > 0$, the Hessian is called -ve definite and the function are concave and the function has a maximum value (E T Dowling 1986).

The idea of convex sets and convex/concave functions has many important properties that are very useful techniques applied in economics particularly in various optimizing problems. Suppose f(x) is concave then - f(x) will be convex i.e. one is the mirror image of the other. Thus minimization of one will maximize the other and hence it is used in case of duality. As a result if we analyse the properties and applications of convexity the other is automatically done. To find global/local maximum/minimum and in duality the idea of convexity is very much helpful. The convexity property is the heart of consumer preference theory and in production techniques.

Convexity property under uncertain behaviour

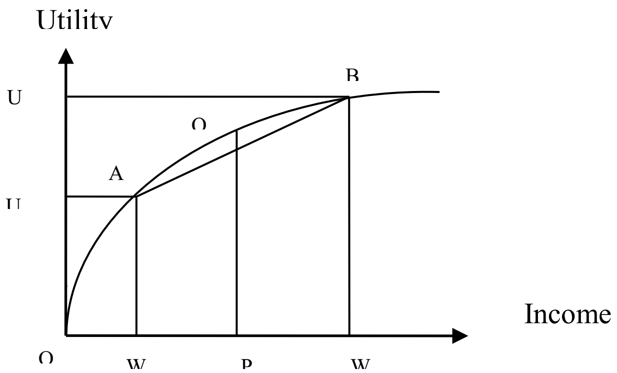
For the sake of simplicity here we assume that function can be measured the return on gambles on monitory terms, is strictly increasing and is continuous with first and second order derivatives exist. In this situation the consumer's expected utility of a gamble: $E[W] = pW_1 + (1-p)W_2$ where $W_i, \forall i = 1,2$ are different wealth levels and p is the probability of outcome of lottery ($0<p<1$) (J Henderson and R E Quandt 1987).

Now the person is risk neutral if the expected value of the lottery equals the expected utility of value of the lottery i.e.

$$U(pW_1 + (1-p)W_2) = p\,U(W_1) + (1-p)\,U(W_2)$$

In this situation expected utility function will be 45^0 line i.e. he/she has no preference either in gambling or certainty.

Now if the consumer is risk averse then the expected utility function will be concave. If we take any two points A and B on the expected utility function then the chord must lie above the joining line $\overline{AB}$ (as shown in the following figure).



$$U(pW_1 + (1-p)W_2) > p\,U(W_1) + (1-p)\,U(W_2)$$

Thus we can say that concave utility function assured risk aversion. In this situation the consumer acceptance set A (W) must be convex (A (W) be the set of all gambles the consumer would accept at an initial wealth level W).

Now suppose the consumer is risk lover or plunger. Here his/her preference directly follows the Charbak principle i.e. his/her preference are such as that today's pigeon is greater than tomorrow's peacock. In this case the expected utility function will be convex. Then the chord must lie below the joining line AB. Here the utility of expected value is less than its expected utility i.e.

$$U\ (pW_1 + (1-p)\ W_2) < p\ U\ (W_1) + (1-p)\ U\ (W_2)$$

Arrow-Pratt used the following formula to measure the absolute risk aversion:

$$r(W) = -\frac{r''(W)}{r'(W)}$$

Now if r (W) <0 implies the utility function is strictly concave and the consumer is risk averter. And if r (W) >0 implies the utility function is strictly convex and the consumer is risk lover.

Convexity and Production

The convex production set similarly draw the CRS or DRS. In general equilibrium models only allow the non-convex production sets (or economics of scale) or non convex preferences. Production function is a purely technical relation which connects factor inputs and outputs (A Koutsoyiannis 1979). A method of production activity is a combination of factor inputs require for the production of one unit of output.

Suppose the producing firm has the following activity process:

	P_1	P_2	P_3
Labour	2	0	3
Capital	2	3	0

In the first process the firm is using a convex technology since here the combination of both the factors are used while in the other two cases only single output is used.

Minkowski addition of sets

Let $Q_1 = [0, 1]^2$, $Q_2 = [1, 2]^2$, $Q_3 = [1, 3]^2$

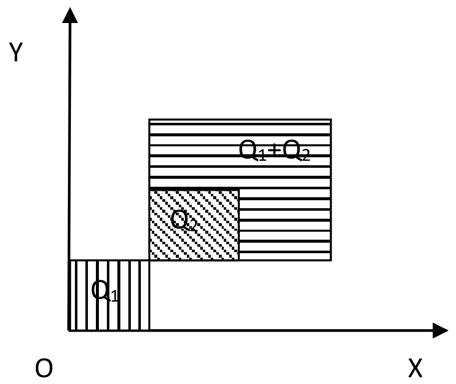
Then $Q_3^2 = Q_1^2 + Q_2^2$

From Minkowski addition of sets we get the sum of the squares Q_1^2 and Q_2^2 is the square of $Q_1 + Q_2$ i.e. $Q_1^2 + Q_2^2 = (Q_1 + Q_2)^2$

Suppose a firm has the following production process:

	Lab. Cap.	Lab. Cap.	Lab. Cap.
Q_1	[0, 1]	[1, 2]	[2, 3]
Q_2	[1, 2]	[2, 3]	[3, 4]
Q_3	[1, 3]	[3, 5]	[5, 7]

Activity Q_3 is derived from Q_1 and Q_2 i.e. simply adding Q_1 and Q_2.
In production process $Q^2_{L+K} = Q_L^2 + Q_K^2$



Though the cost of productions in $Q_1 + Q_2$ is the same in Q_3 it is better to use the activity Q_3 to get the advantage of large scale production process. Here Q_1 may be called small-scale production process, Q_2 may be called medium-scale production process Q_3 may be called large-scale production process. By means of the combination of Q_1 and Q_2, Q_3 will be a possible and efficient method using increasing returns to scale.

The idea of convexity may be used in resource allocation in production process. Suppose we consider two production functions as:

$Q_1 = f_1(K_1, L_1)$ and $Q_2 = f_2(K_2, L_2)$ which are non-decreasing, differentiable and concave.

Then the aggregate production functions can be written as:

$Q = Q_1 + Q_2 = \emptyset(K, L)$

Let aggregate capital= K and aggregate labour= L.

Then $K_1 + K_2 \leq K$ & Then $L_1 + L_2 \leq L$

The aggregate production can be obtained as follows:

$\emptyset(K, L) = max.\{ f_1(K_1, L_1) + f_2(K_2, L_2): K_1 + K_2 \leq K, L_1 + L_2 \leq L, L, K \geq 0\}$

We know that the sum of two convex sets must be convex. Suppose f_1 (K_1, L_1) and f_2 (K_2, L_2) are homogenous and show constant return to scale (CRS). Then their convex combination must be convex and shows CRS.

Let $\emptyset(K, L)$ = t f_1 (K_1, L_1) + (1-t) f_2 (K_2, L_2)$\forall t$, 0<t<1.

Q_1= f_1 (K_1, L_1) =$K_1{}^a L_1{}^b$ and Q_2= f_2 (K_2, L_2) = $K_2{}^c L_2{}^d$

Now let Q_{11}= f_1 (K_1, L_1) = log x and Q_{22} = f_2 (K_2, L_2) = log y

Since $f_1'(x) = \frac{1}{x} > 0$ *and* $f_2'(y) = \frac{1}{y} > 0$

∴ Q_{11} and Q_{22} are homothetic function.

Suppose ϕ= Q_{11} + Q_{22} = $\emptyset(K, L)$ = t x + (1-t) y

It should be noted that $\emptyset(K, L)$ is not homogenous but homothetic.

Thus ϕ = t x + (1-t) y = t log x + (1-t) log y = log $x^t y^{1-t}$ = log z, say.

Now $f/(z)$= $\frac{1}{z} > 0$ Thus again ϕ is homothetic.

Result: The sum of the MPs of each times the level of use of that factor times the level of use of that factor is the convex combination of the share of input elasticities.

Proof: ϕ = t x + (1-t) y = t log x + (1-t) log y = log $x^t y^{1-t}$ = log $(K_1{}^a L_1{}^b)^t (K_2{}^c L_2{}^d)^{1-t}$

= t log$K_1{}^a L_1{}^b$ + (1-t) log $(K_2{}^c L_2{}^d)$

∴ $K_1\ \phi_{K_1} + L_1\ \phi_{L_1} + K_2\ \phi_{K_2} + L_2\ \phi_{K_{L_2}}$ = t a + t b + (1-t) c + (1-t) d

= t (a + b) + (1-t) (c + d)

Let a + b = ρ and c + d = σ

∴ $K_1\ \phi_{K_1} + L_1\ \phi_{L_1} + K_2\ \phi_{K_2} + L_2\ \phi_{K_{L_2}} = t$ ρ + (1-t) σ

Hence the result QED.

Now we turn to our main objective function:

$\emptyset(K, L) = max.${ f_1 (K_1, L_1) + f_2 (K_2, L_2): $K_1 + K_2 \leq K$, $L_1 + L_2 \leq L$, L, K≥ 0}

Substituting the values we get,

$\emptyset =$ t log$K_1{}^a L_1{}^b$ + (1-t) log $(K_2{}^c L_2{}^d)$ + μ [L - (L_1+ L_2+s_1)] + θ[K - (K_1+ K_2+s_1)] , where s_1& s_2 are dummy variables.

Here we apply the Kuhn-Tucker conditions as follows:

$\frac{\partial \emptyset}{\partial K_1} = \frac{bt}{L_1} - \mu \geq 0$ *and* $L_1 . \frac{\partial \emptyset}{\partial K_1} = 0$

∴ $L_1 \left(\frac{bt}{L_1} - \mu\right) = 0$. But $L_1 \neq 0$ ∴ $L_1 = \frac{bt}{\mu}$

lly, lly $K_1 = \frac{bt}{\theta}$, $L_2 = \frac{(1-t)d}{\mu}$ *and* $K_2 = \frac{(1-t)c}{\theta}$

∴ $L^{*}= L_1 + L_1 = \frac{1}{\mu}(b\, t + (1\text{-}t)\, d)$

$K^{*}= K_1 + K_1 = \frac{1}{\theta}(a\, t + (1\text{-}t)\, c)$

Now the Lagrangian constants θ and μ are shadow prices of that particular resource use in the production process.

Lemma: Optimum amount of a particular resource used is the per unit shadow price of the convex combination of the share of input elasticities.

Proof: We have already got,

$L^{*} = \frac{1}{\mu}(b\, t + (1\text{-}t)\, d)$ and $K^{*}= \frac{1}{\theta}(a\, t + (1\text{-}t)\, c)$.

-which follows the result directly QED.

Result: $\emptyset$ *is strictly concave:*

Proof: $Q_{1L_1} = \frac{b}{L_1}$ and $Q_{1L_1L_1} = -\frac{b}{{L_1}^2}$

$Q_{1K_1} = \frac{a}{K_1}$ and $Q_{1K_2K_2} = -\frac{a}{{K_1}^2}$

$Q_{1L_1K_1} = Q_{1K_1L_1} = 0$

∴ $Q_{1L_1L_1} = -\frac{b}{{L_1}^2} < 0,$

$Q_{1K_2K_2} = -\frac{a}{{K_1}^2} < 0 \text{ and } Q_{1L_1L_1} . Q_{1K_2K_2} - Q_{1L_1K_1} Q_{1K_1L_1} > 0.$

Here $|H_1| > o \text{ and } |H_2| > 0$, the Hessian is called +ve definite and the function $\emptyset$ is strictly convex

lly, lly $Q_2$is strictly concave.

Since $\emptyset$ is derived by homothetic transformation (log transformation) Q_1 & Q_2 are strictly concave, therefore $\emptyset$ *is also strictly concave.*

Hence the result QED.

Lemma: $\emptyset$ *is homothetic but not homogeneous.*

Proof: By definition ϕ = t x + (1-t) y = t log x + (1-t) log y = log $x^t y^{1-t}$ = log z, say.

Now ϕ / (z) = $\frac{1}{z} > 0$

This directly implies ϕ is homothetic.

Result: If Q_1 & Q_2 follow CRS then ϕ also follows CRS.

Proof: Suppose f_1 (K_1, L_1) and f_2 (K_2, L_2) are homogenous and show constant return to scale (CRS). Then a + b = 1 and c + d =1 and their convex combination $\emptyset(K, L)$ = t f_1 (K_1, L_1) + (1-t) f_2 (K_2, L_2)$\forall t$, 0<t<1follows CRS.

$K_1\ \phi_{K_1} + L_1\ \phi_{L_1} + K_2\ \phi_{K_2} + L_2\ \phi_{K_{L_2}}$ = t a + t b + (1-t) c + (1-t) d

= t (a + b) + (1-t) (c + d) =1, substituting the values of (a + b) & (c + d).

Hence the result QED.

Conclusion

There are now more than a billion malnourished people in the world meaning that almost one sixth of humanity is suffering from hunger. Fertility of land has been reduced due to over exploitation, excessive use of chemical fertilisers, insecticides and pesticides. Due to indiscriminate deforestation the amount of rainfall reduces and land erosion takes place. Natural calamities like droughts, floods, cyclones, global warming, melting glaciers, raising sea level etc. are increasing and environment is degraded. As a result the production of food grains decreases and made deprivation at a global level. Not only that sometimes food crisis arises not due to food shortage, there is enough food in the stock though some people are in starvation due to some failure of public distribution system. The crisis not food crisis, it is the policy crisis of the government. This is a tragedy from food surplus to food scarcity to deprivation.

In consumer behaviour theory we found that MRS decreases along an IC and MRS decreases in absolute term iff preferences are strictly convex. We do not have diminishing MRS with downward slopping IC with non-convex preference. In the theory of production by means of homothetic transformation we get that the sum of the MPs of each times the level of use of that factor times the level of use of that factor is the convex combination of the share of input elasticities. The Lagrangian constants θ and μ are shadow prices of that particular resource use in the production process. The firm may take decision regarding optimum amount of a particular resource used is the per unit shadow price of the convex combination of the share of input elasticities. The main cause of deprivation is the over or under consumption and over or under production of any commodity or a particular resource. By choosing the value of parameters like a, b, c, d, θ and μ we can simply allocate the resources for production and distribute the goods according to the common needs. This will lead to a better welfare state and obviously reduce the deprivation, hunger, poverty which is the common phenomena of the rural masses.

Acknowledgement: *In preparation of this paper I deeply acknowledge my respected sir Prof. Jaydeb Sarkhel, Department of Commerce, Burdwan University, West Bengal, India, e-mail: jaydebsarkhel@gmail.com*

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