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Reliability computation technique for I-beam under the Gamma setup

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Abstract

Stress function of I-beam is function of multiple stochastic factors and this system is so complex that exact analytical determination for reliability is difficult to obtain. To address this pressing problem, in this article, we have made an attempt to approximate system reliability of this crucial item based on reliability bounds under the Gamma setup. This article also provides level of error of this item. Numerical analysis has been under taken to show the adjacent between the upper and lower bounds of this item.

Keywords: Gamma distribution, I-beam, Reliability values and Extent of error, Stress - Strength analysis, System Reliability

INTRODUCTION

The word ‘Reliability’ refers to the ability of a system to perform its stated purpose adequately for a specified period of time under the operational conditions encountered (Gertsbakh, 1989). Any system will be absolutely reliable if some undesirable events, called failures, do not occur in the system operation.

Reliability of a system (Kapur and Lambersion, 1976) can be either studied in terms of the system life or examined in terms of the stress-strength relationship. Under system life, system reliability is the probability of survival of the system for the predefined mission time.

Under the stress- strength model, both stress and strength are considered to be random variables and reliability is defined as the chance the stress is less than strength. If stress is more than the strength, the system fails to operate. If X is the strength and S is the stress then Reliability, R, is defined as

$$R = P(X-S>0) \text{ i.e. } P(X>S)$$

Exact analytical computation of reliability of complex engineering item is very difficult task because complex engineering items are functions of multiple stochastic factors.

PROBLEM IN PRECISE ANALYTICAL DETERMINATION OF RELIABILITY

Under the direct knowledge of stress-strength distributions, ordinary transformation technique due to Parzen (1960) can be used for determination of system reliability. Mostly problem arises in determination of reliability for many engineering items when the stress or strength variables are themselves functions of multiple stochastic factors. According to English et al (1996) analytical approaches are intractable, if not impossible, for most reasonably sized problems. The inference theory approach for reliability prediction of Kapur and Lamberson (1976) Sheth and et al (1968), also fails because for complex system strength or stress distributions are not mostly known.

We observe from the literature that determining the system reliability under stress-strength model, even for a simple engineering system with continuous distribution, presents derivational difficulties. For example, if we consider solid-shaft, a very common engineering item, we fail to obtain reliability value in a closed analytical form under non-normal setup. The problem remains intractable also for non-normal setup like the Exponential, Weibull and other widely used non-normal setups. If we want to determine reliability of a hollow cylinder under stress-strength model, then the task becomes the more difficult one.

SEVERAL TECHNIQUES OF SOLUTION

When accurate solution is not obtainable, some substitute techniques must be adopted to arrive at a close approximation of the actual reliability. There are such techniques have been reported in the literature for approximating the distribution of a complex system. In the literature, such methods are (1) Discrete concentration approach (2) Moment equalization method (3) Failure rate (FR) equalization (4) Mean residual life (MRL) equalization (5)Other methods like range approximation (6) Reliability approximation technique.

EARLIER WORKS

D`Errico and Zaino (1988) employed Taguchi`s (1978) concept of experimental design and they presented discretization approach for approximating the behaviour of complex system. Later English et al (1996) elaborated the same approach by enhancing the number of discrete point. They compared discrete approximation with simulated values. But their study was of course limited to normal setup. M. Xie and C.D. Lai (1998) presented approximation of system reliability using one step conditioning. Jianan Xue and Kai Yang (1997) setup stress-strength inference reliability model with strength degradation under the assumptions that stress-strength are statistically independent. They also found simple formulae for estimating upper and lower bounds for stress-strength reliability. The concept of discrete concentration of Roy (1993) was employed by Roy and Dasgupta (2001) for presenting discretizing procedure. Roy (2002, 2003) examined in details discrete normal and discrete Rayleigh Distribution in this process.

Roy (2004) introduced the concept of linear transformation of the discretized variable for equalization of the first two moments of a continuous variable. He examined applicability of discretization approach through studies on shear stress of different engineering items. Roy and Dasgupta (2002) examined discretization of Weibull distribution and Roy (2004) presented discretization of Rayleigh distribution to approximate the system reliability. Roy (2002) also considered discretization of the uniform distribution. Roy (2005) discussed Characterization of bivariate discrete distributions based on mean residual life properties. Roy and Ghosh (2009) studied discretization of continuous random variable applying the failure rate function to approximate the system reliability. In particular, they studied the discretization of the Rayleigh and Lomax distributions. Barbiero (2010) considered a discretizing method of reliability computation in complex system under stress-strength model. Recently, reliability approximation under stress strength model from different consideration has been examined by Nayak (2011). Nayak and Roy (2012) proposed bound based reliability approximation under the Weibull, Rayleigh and Exponential setups. Nayak and Roy (2012) also studied a new approach for approximating reliability of a complex system under the Weibull setup. Nayak and Roy (2012) also examined bound based reliability approximation of an engineering item, Resistor, under the stress-strength model. Nayek et al (2014) has approximated reliability of solid shaft under the Gamma setup.

Ghosh et al (2013) proposed reliability approximation through the discretization of random variables using reversed hazard rate function. Using one step conditioning, Xie and Lai (1998) examined approximation of system reliability. Under the assumptions that stress-strength are statistically independent. Xue and Yang (1997) studied a stress-strength inference reliability model with strength degradation. They also presented simple formulas for estimating upper and lower bounds for stress-strength reliability. Kundu and Gupta (2005) considered estimation of P(X>S), where X and S are treated as independent random variables.

However, these studies were restricted to the discrete concentration approach only. Bound-based reliability approximation may be a better alternative as it has wider application than the discrete concentration approach (Roy, 1993).

The purpose of this paper is to approximate reliability of complex system based on reliability bounds. Here we will consider the case where X and S are independent Gamma random variables. Interesting feature of the proposed approach is that the reliability approximation comes out as a function of distributional parameters and it can be adjusted for designing and redesigning the system to ensure the maximum level of reliability at a given cost per requirements.

But discrete approximation for any engineering item cannot adjust in terms of the parameter of the distribution. Reliability bounds on the other hand are functions of distributional parameters. Keeping these issues in mind, we have approximated reliability based on reliability bounds of the simplified form of I-beam under the Gamma setup.

RELIABILITY BOUNDS

If we can find close lower and upper bounds of reliability, the determination of system reliability becomes considerably easier as pointed out in Gertsbakh (1989). Usually these bounds are found to be satisfactory for practical purposes. Further, the computation of exact reliabilities are difficult than the computation of reliability bounds. We have already pointed out that reliability bounds can be obtained in terms of design parameters and these can be adjusted appropriately.

As reliability bounds are item dependent and setup dependent, we have considered a very common but an crucial engineering item for computing the reliability bounds under the Gamma setup.

Gamma distribution is a very effective model for reliability analysis and can give rise to other well-known models under several parametric choices. Gamma distribution has extensive applicability. So, a study on reliability approximation on Gamma frame work will have a wider appeal.

The shear stress of modified form of I-beam is given by (Kapur and Lamberson, 1976)

$$S = \frac{M}{.0822D^3},$$

where, S, M and D are shear-stress, maximum moment at the point of application of load and width of the I-beam, respectively.

RELIABILITY BOUNDS UNDER THE GAMMA SETUP

We assume that M follows G (β, n) and D follows G $(\boldsymbol{\theta}$, r). Let the built in strength X follows yet another G (α, t). We also assume that X, M and D are statistically independent.

We now consider the following two lemmas to get reliability upper and lower bounds.

Lemma 6.1: If X is random variable such that $E(e^{cx})$ exists for c > 0 then $P(X > y) \leq \frac{E(e^{ax})}{e^{ay}}$

Lemma 6.2: If F $\in \mathrm{IFR}$ and E (T) = ρ then for $0 \leq \mathrm{y} \leq \rho$, $R \geq \exp(-\frac{\mathrm{y}}{\rho})$

Result 6.1: If the strength and the stress component random variables of the I-beam follow Gamma distributions then upper bound for the system reliability, R, is given by

$$R_U(\alpha, c, n, \beta, \boldsymbol{\theta}, r, y) = \left(\frac{\alpha}{\alpha-c}\right)^y\{1-\frac{cn\theta^3\Gamma(r-3)}{.0822\beta\Gamma(r)}+\frac{n(n+1)c^2\theta^6\Gamma(r-6)}{1.3415\beta^2\Gamma(r)}\}, \quad \text{for } r>6$$

Proof: According to the definition of reliability under the stress strength model,

$$R=P(X-S>0), \text{ where } S=\frac{M}{.0822D^3}$$

Now from lemma1, we note that

$$R = P(X>s_{|S=s}) \le \frac{E(e^{cX})}{e^{cs}}$$

$$= E_A E_M\left[\frac{\left(\frac{\alpha}{\alpha-c}\right)^y}{e^{\alpha\frac{M}{.0822D^3}}}\right]$$

$$= \int_0^\infty\int_0^\infty \left(\frac{\alpha}{\alpha-c}\right)^y e^{-c\frac{M}{.0822D^3}}\, dF_M dF_D \qquad \text{.......... (i)}$$

Note that $e^{-X} \le 1-X+\frac{X^2}{2}$, for X >0

Therefore (i) can be written as

$$R\le \left(\frac{\alpha}{\alpha-c}\right)^y\int_0^\infty\int_0^\infty\{1-\frac{cM}{.0822D^3}+\frac{c^2M^2}{2\times(.0822)^2D^6}\}\, dF_M dF_D$$

$$= \left(\frac{\alpha}{\alpha-c}\right)^y\{1-\frac{cE(M)E\left(\frac{1}{D^3}\right)}{.0822}+\frac{c^2E(M^2)E\left(\frac{1}{D^6}\right)}{.01351}\}$$

$$= \left(\frac{\alpha}{\alpha-c}\right)^y\{1-\frac{cn\theta^3\Gamma(r-3)}{.0822\beta\Gamma(r)}+\frac{n(n+1)c^2\theta^6\Gamma(r-6)}{.01351\beta^2\Gamma(r)}\}, \text{ for } r>6 \qquad \text{.......... (ii)}$$

Hence, upper bound for the system reliability, R, when stress and strength components of I-beam follow gamma distribution, is given by

$$R_U(\alpha, c, n, \beta, \boldsymbol{\theta}, r, y) = \left(\frac{\alpha}{\alpha-c}\right)^y\{1-\frac{cn\theta^3\Gamma(r-3)}{.0822\beta\Gamma(r)}+\frac{n(n+1)c^2\theta^6\Gamma(r-6)}{.01351\beta^2\Gamma(r)}\}, \text{ for } r>6.$$

Result 6.2: If the strength and the stress component random variable of the I-beam follows Gamma distribution then lower bound for the system reliability, R, is given by

$$R_L(\alpha, t, n, \beta, \boldsymbol{\theta}, r, y) = 1-\frac{\alpha n\theta^3\Gamma(r-3)}{.0822t\beta\Gamma(r)}, \text{ for } r>3.$$

Proof: Using lemma2, $R \ge \exp\left(-\frac{S}{\mu_x}\right)$

Therefore,

$$R \ge \exp\left(-\frac{\alpha S}{t}\right)$$

$$= exp\left(-\frac{\alpha}{t}\frac{M}{.0822D^3}\right)$$

$= E_A\, E_M(e^{-\frac{\mathrm{M}}{.0822\mathrm{D}^3}\frac{\alpha}{t}})$

$= \int_0^{\infty}\int_0^{\infty}(e^{-\frac{\mathrm{M}}{.0822\mathrm{D}^3}\frac{\alpha}{t}})\mathrm{dF_M dF_D}$ (iii)

Note that $e^{-X} \geq 1 - X$, for $X > 0$

Therefore (iii) written as

$= \int_0^{\infty}\int_0^{\infty}[1 - \frac{\mathrm{M}}{.0822\mathrm{D}^3}\frac{\alpha}{\mathrm{t}}]\,\mathrm{dF_M dF_D}$

$= 1 - \frac{\alpha \mathrm{E(M)E}(\frac{1}{\mathrm{D}^3})}{0.822\mathrm{t}}$

$= 1 - \frac{\alpha n\theta^3\Gamma(r-3)}{.0822 t\beta\Gamma(r)}$, for $r > 3$. (iv)

Hence, Lower bound for the system reliability, R, when stress and strength components of I-beam follow gamma distribution, is given by

$\mathrm{R_L}$ (α, t, n, β, **θ**, r, y) = $1 - \frac{\alpha n\theta^3\Gamma(r-3)}{.0822 t\beta\Gamma(r)}$, for $r > 3$.

RELIABILITY VALUES AND ERROR TERMS

Here we suggest mean of two bounds as the reliability approximation and half of the absolute difference between the two bounds as the extent of error. It will be obtained as function of distributional parameters. So, this work is very crucial to the reliability engineers because they will be able to enhance or reduce reliability according to their requirement.

Reliability approximation is given by

$$\mathrm{R_A}(\alpha, t, n, \beta, \boldsymbol{\theta}, r, c) = \frac{\mathrm{R_U}\,(\alpha,c,n,\beta,\theta,r) + \mathrm{R_L}\,(\alpha,t,n,\beta,\theta,r)}{2}$$

$$= \frac{(\frac{\alpha}{\alpha-c})^{y}\left\{1 - \frac{cn\theta^3\Gamma(r-3)}{.0822\beta\Gamma(r)} + \frac{n(n+1)c^2\theta^6\Gamma(r-6)}{1.3415\beta^2\Gamma(r)}\right\} + \{1 - \frac{\alpha n\theta^3\Gamma(r-3)}{.0822t\beta\Gamma(r)}\}}{2} \qquad \text{(v)}$$

This is the reliability approximation in terms of distributional parameters for I-beam when stress and strength components follow gamma distribution and extent of error in terms of distributional parameters is given by

$$\mathrm{R_E}(\alpha, t, n, \beta, \boldsymbol{\theta}, r, c) = \left|\frac{\mathrm{R_U}\,(\alpha,c,n,\beta,\theta,r) - \mathrm{R_L}\,(\alpha,t,n,\beta,\theta,r)}{2}\right|$$

$$= \left|\frac{(\frac{\alpha}{\alpha-c})^{y}\left\{1 - \frac{cn\theta^3\Gamma(r-3)}{.0822\beta\Gamma(r)} + \frac{n(n+1)c^2\theta^6\Gamma(r-6)}{1.3415\beta^2\Gamma(r)}\right\} - \{1 - \frac{\alpha n\theta^3\Gamma(r-3)}{.0822t\beta\Gamma(r)}\}}{2}\right| \qquad \text{(vi)}$$

This is the extent of error in terms of distributional parameters for I-beam when stress and strength components follow gamma distribution.

EXAMINE OF ADJACENT BETWEEN THE RELIABILITY BOUNDS OF I-BEAM

We now find the reliability values and extent of error for this important complex engineering item. For this reason, we have determined lower and upper reliability bounds for some specific choices of the distributional parameters of the I-beam. The specific choices of distributional parameters, considered here in, are y = 85, c =.00001, $\alpha = 2000$, $\theta = 5$, $n = 80$, $r = 90$. We allow the other parameter, β to vary so that it can cover a wide range of reliability values. The corresponding reliability values and extent of error have been shown in the table8.1. From this table we observe that reliability upper and lower bounds for this engineering item are very close to each other. Therefore, we can propose average of the two bounds as the reliability values for this important engineering item and half of the absolute deviation as the extent of error. It may be noted from the given table that error term sharply reduces as reliability enhances.

Table8.1:
Reliability values and Extent of error under the gamma setup of I-beam.

Sl. No.	Stress parameter (β)	Reliability Values	Extent of Error
1	10.0	0.7899527	2.100475e-01
2	11.0	0.8090479	1.909523e-01
3	12.0	0.8249606	1.750396e-01
4	13.0	0.8384252	1.615751e-01
5	14.0	0.8499663	1.500340e-01
6	15.0	0.8599685	1.400318e-01
7	16.0	0.8687205	1.312798e-01
8	17.0	0.8764429	1.235575e-01
9	19.0	0.8894489	1.105514e-01
10	20.0	0.8949765	1.050239e-01
11	23.0	0.9086752	9.132513e-02
12	25.0	0.9159812	8.401914e-02
13	28.0	0.9249832	7.501711e-02
14	30.0	0.9299844	7.001599e-02
15	35.0	0.9399866	6.001373e-02
16	40.5	0.9481366	5.186375e-02
17	60.0	0.9649923	3.500810e-02
18	80.0	0.9737443	2.625613e-02
19	100.0	0.9789955	2.100494e-02
20	150.	0 0.9859970	1.400337e-02
21	200.0	0.9894978	1.050258e-02
22	500.0	0.9957993	4.201159e-03
23	700.0	0.9969995	3.000888e-03
24	900.0	0.9976664	2.334072e-03
25	1000	0.9978997	2.100686e-03
26	1500.0	0.9985999	1.400528e-03
27	2000.	0 0.9989500	1.050449e-03
28	10000.0	0.9997902	2.102598e-04
29	20000.0	0.9998952	1.052362e-04
30	300000.0	0.9999932	7.214077e-06
31	400000.0	0.9999950	5.463683e-06

CONCLUSIONS

For a complex system, precise analytical determination of reliability is mostly intractable. From the literature, we observe that different authors have proposed different numerical procedures to approximate system reliability for intractable cases. In the literature, there is no work for reliability approximation with extent of error in terms of distributional parameters. So, further manipulation in terms of distributional parameters cannot be under taken using subsisting approaches.

So, to bridge this gap in the literature, we have handled the difficult analytic reliability determination problem in terms of reliability bounds. Our proposed reliability approximation based on midpoint of the bounds is a function of distributional parameters. So, one can enhance or reduce reliability according to their requirement.

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